Table 1 Values of energy efficiency, e_R

α/γ	1.1	1.25	1.40	1.67
0.0	0.770774	0.981932	0.996722	0.959639
0.00005	0.764761	0.981826	0.996853	0.961653
0.0001	0.760494	0.980106	0.998962	0.963174
0.0005	0.612748	0.968819	1.007108	0.977321
0.001	0.341371	0.929104	1.025500	0.997332

shock conditions (5) as the initial conditions. With the help of the solution so obtained and Simpson's formula, the energy efficiency function e_R is evaluated as

$$e_R = \frac{1}{\beta}\,\frac{I_2^2}{I_3}$$

where

$$I_2 = \int_0^1 f(h^2+\gamma g/f)^{1/2}\eta^2 d\eta$$

$$I_3 = \int_0^1 (fh^2+\gamma g)\eta^2 d\eta$$

The functions e_R is tabulated in Table 1 for different values of α and γ. For larger γ, the energy efficiency (for $\alpha=0$) decreases, as can be seen from Table 1. This fact is difficult to notice in the graphic representation of e_R in Ref. 1.

It is interesting to note that with increasing α, the energy efficiency decreases for lower γ while it increases for larger γ. Also for $0 \leq \alpha \leq 0.001$, the functional value of I_2 is affected by about 6-10%. However, for $\alpha > 0.001$, certain inconsistencies are observed during computation. Finally, the time t_s^* at which a point source again allows a finite mass flux is further reduced for increasing α and for all values of the coefficient of specific heat.

Acknowledgment

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Technical Comments

Comment on "High-Frequency Subsonic Flow Past a Pulsating Thin Airfoil"

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PLOTKIN[1] developed high-frequency approximation for subsonic potential flow past a nonlifting pulsating airfoil. The method of solution given by Plotkin[1] requires a great deal of labor and is given as a Green's function source distribution along the chordline of the airfoil involving an asymptotic evaluation of the source integral using the method of stationary phase for the perturbation velocity potential. The purpose of this Note is to show that the problem in question lends itself so readily to the Laplace transform method that the development of the solution becomes almost trivial.

One has, for the linearized two-dimensional unsteady subsonic potential flow of an otherwise uniform stream with speed U, Mach number M, and speed of sound a past a thin nonlifting pulsating airfoil of chord l and thickness-to-chord ratio of $O(\epsilon)$,

$$(1-M^2)\phi_{xx}+\phi_{yy}-(2Mi\omega/a)\phi_x+(\omega^2/a^2)\phi=0 \tag{1}$$

where the velocity potential is given by

$$\Phi(x,y,t)=Ux+\epsilon\phi(x,y)e^{i\omega t} \tag{2}$$

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and the x axis is directed along the freestream and the chordline and the origin is at the leading edge.

If the surface of the pulsating airfoil is given by

$$f(x,t)=g(x)e^{i\omega t} \tag{3}$$

one has the linearized body boundary condition in the high-frequency approximation

$$\phi_y(x,0^\pm)=\pm i\omega g(x) \tag{4}$$

Upon taking the Laplace transform of Eqs. (1) and (4), as defined by

$$\tilde{\phi}(s,y)=\int_0^\infty e^{-sx}\phi(x,y)dx \tag{5}$$

one obtains

$$\tilde{\phi}_{yy}+\mu^2\tilde{\phi}=0 \tag{6a}$$

$$\tilde{\phi}_y(s,0^\pm)=\pm i\omega\tilde{g}(s) \tag{6b}$$

where

$$\mu^2=[(1-M^2)s^2-(2Mi\omega/a)s+(\omega^2/a^2)]$$

and, in the high-frequency approximation,

$$\mu \approx (\omega/a)-iMs$$

From Eq. (6), one obtains

$$\tilde{\phi}(s,y)=\frac{\omega\tilde{g}(s)e^{-i\mu|y|}}{\mu} \tag{7}$$

and, in the high-frequency approximation,

$$\tilde{\phi}(s,y)\approx -ae^{-i\omega|y|/a}\tilde{g}(s)e^{-M|y|s} \tag{8}$$

Upon inverting, one obtains

$$\phi(x,y) = \begin{cases} -ag(x-M|y|)e^{-i\omega|y|/a}, & x>M|y| \\ 0, & x<M|y| \end{cases} \tag{9}$$

which explains why the perturbation is nonzero only within a region bounded by the rays $x=M|y|$ from the leading edge and $x-l=M|y|$ from the trailing edge; this provides a posteriori the raison d'etre for the Laplace transformation of Eqs. (1) and (4).

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Comment on "Numerical Solutions of the Compressible Hodograph Equation"

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THE particular problem of the ideal two-dimensional compressible gas jet issuing freely from a reservoir under pressure to ambient surroundings has been of signal importance to the history of fluid dynamics and found its revealing solution in a classical paper by Chaplygin.[1] This problem has now been used by Liu and Chow[2] to test a useful proposal which they have put forward for the numerical solution of this type of two-dimensional gasdynamic problems, which are initially set in the hodograph plane and can then enjoy the benefit of linearization in the governing differential equations. The problems that can thus be treated are mainly of the inverse variety and therefore offer the analyst the added attraction of a certain latitude in the choice of the boundaries in the hodograph plane, within the framework of general design constraints on the dynamics of the flows investigated. There have been many attempts at fashioning a pragmatic and dependable engineering tool based on certain approximations that have been found more or less acceptable, depending on the degree of accuracy demanded of the solution and/or the particular aspect of the solution sought. From one such attempt,[3] the author has presented in Ref. 4 a good engineering solution to the problem of the ideal two-dimensional compressible jet issuing from a straight-walled convergent nozzle ($V_a/V_\infty=0$) of arbitrary included angle 2α, from which the following closed-form expression for the contraction coefficient $cc(\alpha, q'_\infty)$ was obtained:

$$cc(\alpha, q'_\infty) = \frac{1}{1+\rho'_\infty K(\alpha)(1-I_{1\infty})}$$

where $(\rho'_\infty, q'_\infty)$ represent the conditions (nondimensionally expressed) of density and velocity on the free-boundary streamline or "at infinity" in the jet (denoted by the suffix ∞), and,

$$K(\alpha) = \frac{1}{\alpha}\int_0^\alpha \cos\left(\frac{\pi\theta}{2\alpha}\right)\sin\theta \, d\theta \qquad I_{1\infty} = \int_\infty^{\Lambda_\infty} \ell n \, T d\Lambda$$

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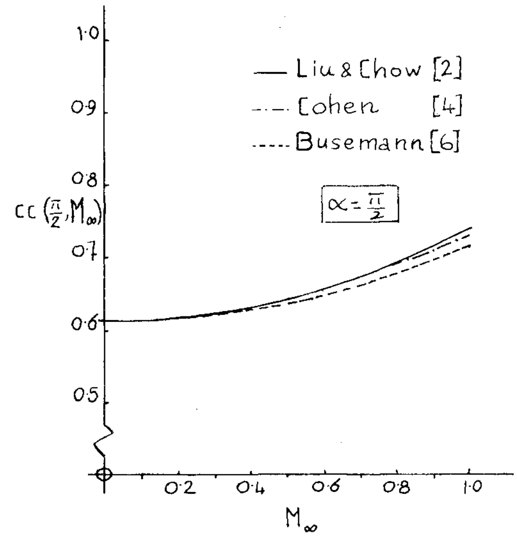

Fig. 1 Dependence of contraction coefficient on M_∞.

where

$$T=\frac{\sqrt{1-M^2}}{\rho'} \quad \text{and} \quad d\Lambda = -\frac{\sqrt{1-M^2}dq'}{q'}$$

Figure 1 shows a comparison of the contraction coefficients for the particular case $\alpha=90$ deg, obtained by the two methods in Refs. 2 and 4, for the range $0<M_\infty\leq 1$ on the free streamline of the jet.

The agreement between the results achieved by the numerical near-exact approach of Liu and Chow[2] and those derived[4] from the engineering-analytic approach of the author[3] is very good, the departure of the latter from the analytically exact given by Chaplygin[1] being less than 2% in defect in the extreme case of the sonic jet ($M_\infty=1$). The method has also been used in Ref. 3 for the design of wind tunnel contractions and turbine/compressor blading with stipulated (and desirable) pressure distributions in compressible subsonic regime (with part-sonic hodograph boundaries allowed).

Finally, an important qualification of a remark by Liu and Chow (Ref. 2, p. 189), "that the final asymptotic state occurs only when x approaches infinity" may be suggested. The remark is generally true, but exceptionally not so, in the case when $M_\infty=1$ (a case apparently examined by these authors). In this extreme case, the condition of parallelism (a sonic throat, here) obtains a finite distance from the nozzle opening, a feature recognized by Chaplygin[1] and confirmed and used by Ovsiannikov.[5]

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